PHY 387K

Advanced Classical Electromagnetism

a graduate level course of lectures given by

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1 Introduction

1.1 Major sources

The textbooks which I have consulted most frequently whilst developing course material are:

Classical electrodynamics: J.D. Jackson, 2nd edition (John Wiley & Sons, New York NY, 1975).

Classical electricity and magnetism: W.K. Panofsky and M. Phillips, 2nd edition, (Addison-Wesley, Reading MA, 1962).

Special relativity: W. Rindler, 2nd edition (Oliver and Boyd, Edinburg and London, 1966).

Foundations of electromagnetic theory: J.R. Reitz and F.J. Milford, 2nd edition (Addison-Wesley, Reading MA, 1967).

Lectures on theoretical physics: A. Sommerfeld, (Academic Press, New York, 1954).

Wave propagation and group velocity: Léon Brillouin, (Academic Press, New York NY, 1960).

Methods of theoretical physics: P.M. Morse and H. Feshbach, (McGraw-Hill, New York NY, 1953).

An introduction to phase-integral methods: J. Heading, (Meuthen & Co., London, 1962).

Radio waves in the ionosphere: K.G. Budden, (Cambridge University Press, Cambridge, 1961).

Classical electromagnetic radiation: M.A. Heald and J.B. Marion, 3rd edition (Saunders College Publishing, Fort Worth TX, 1995).

1.2 Outline of course

You have all presumably taken the standard undergraduate electromagnetism course in which Maxwell's equations are derived and explained. The basic aim of my course is to cover some material which is usually inadequately treated or omitted altogether in undergraduate courses. In fact, I intend to concentrate on three main topics:

1. The relativistically invariant formulation of the laws of electromagnetism.

2. The effect of dielectric and magnetic materials on electric and magnetic fields.

3. The generation, propagation, and scattering of electromagnetic waves.

1.3 The validity of classical electromagnetism

In this course we shall investigate the classical theory of electromagnetism in Euclidian space-time. This theory is valid over a huge range of different conditions, but, nevertheless, breaks down under certain circumstances. On very large length-scales (or close to collapsed objects such as black holes) the theory must be modified to take general relativistic effects into account. On the other hand, the theory breaks down completely on very small length-scales because of quantum effects. It is legitimate to treat a gas of photons as a classical electromagnetic field provided that we only attempt to resolve space-time into elements that contain a great many photons. In conventional applications of electromagnetic theory (*e.g.,* the generation and propagation of radio waves) this is not a particularly onerous constraint.

1.4 Units

In 1960 physicists throughout the world adopted the so-called S.I. system of units, whose standard measures of length, mass, time, and electric charge are the meter, kilogram, second, and coloumb, respectively. Nowadays, the S.I. system is used almost exclusively in most areas of physics. In fact, only one area of physics has proved at all resistant to the adoption of S.I. units, and that, unfortunately, is electromagnetism, where the previous system of units, the so-called Gaussian system, simply refuses to die out. Admittedly, this is mostly an Anglo-Saxon phenomenon; the Gaussian system is most prevalent in the U.S., followed by Britain (although, the Gaussian system is rapidly dying out in Britain under the benign influence of the European Community). One major exception to this rule is astrophysics, where the Gaussian system remains widely used throughout the world. Incidentally, the standard units of length, mass, time, and electric charge in the Gaussian system are the centimeter, gram, second, and statcoloumb, respectively.

You might wonder why anybody would wish to adopt a different set units in electromagnetism to that used in most other branches of physics. The answer is that in the Gaussian system the laws of electromagnetism look a lot "prettier"

than in the S.I. system. There are no ϵ_0 s and μ_0 s in any of the formulae. In fact, in the Gaussian system the only normalizing constant appearing in Maxwell's equations is c, the velocity of light. However, there is a severe price to pay for the aesthetic advantages of the Gaussian system. The standard measures of potential difference and electric current in the S.I. system are the volt and the ampere, respectively. I presume that you all have a fairly good idea how large a voltage 1 volt is, and how large a current 1 ampere is. The standard measures of potential difference and electric current in the Gaussian system are the statvolt and the statampere, respectively. I wonder how many of you have even the slightest idea how large a voltage 1 statvolt is, or how large a current 1 statampere is? Nobody, I bet! Let me tell you: 1 statvolt is 300 volts, and 1 statampere is $1/3 \times 10^{-9}$ amperes. Clearly, these are not particularly convenient units!

In order to decide which system of units we should employ in this course, we essentially have to answer a single question. What is more important to us: that our equations should look pretty, or that the our fundamental units should be sensible? I think that sensible units are of vital importance, especially if we are going to make quantitative calculations (we are!), whereas the prettiness or otherwise of our equations is of marginal concern. For this reason, I intend to use the S.I. system throughout this course.

If, unaccountably, you prefer the Gaussian system of units, there is no reason to despair. Converting formulae from the S.I. system to the Gaussian system is trivial: just use the following transformation

$$\epsilon_0 \rightarrow \frac{1}{4\pi}, \tag{1.1a}$$

$$\mu_0 \rightarrow \frac{4\pi}{c^2}, \tag{1.1b}$$

$$B \rightarrow \frac{B}{c}. \tag{1.1c}$$

The transformation (1.1c) also applies to quantities which are directly related to magnetic field strength, such as the vector potential. Unfortunately, converting formulae from the Gaussian system to the S.I. system is far less straightforward.

As an example of this, consider Coulomb's law in S.I. units:

$$\boldsymbol{f}_2 = \frac{q_1 q_2}{4\pi\epsilon_0}\,\frac{\boldsymbol{r}_2 - \boldsymbol{r}_1}{|\boldsymbol{r}_2 - \boldsymbol{r}_1|^3}. \tag{1.2}$$

Employing the above transformation, this formula converts to

$$\boldsymbol{f}_2 = q_1 q_2\,\frac{\boldsymbol{r}_2 - \boldsymbol{r}_1}{|\boldsymbol{r}_2 - \boldsymbol{r}_1|^3} \tag{1.3}$$

in Gaussian units. However, applying the inverse transformation is problematic. In Eq. (1.3) the geometric 4π in the S.I. formula has canceled with the $1/4\pi$ obtained from transforming ϵ_0 to give unity. It is not at all obvious that the reverse transformation should generate a factor $4\pi\epsilon_0$ in the denominator. In fact, the only foolproof way of transforming Eq. (1.3) back into Eq. (1.2) is to use dimensional analysis. This is another good reason for not using the Gaussian system.

There are four fundamental quantities in electrodynamics; mass, length, time, and charge, denoted M, L, T, and Q, respectively. Each of these quantities has its own particular units, since mass, length, time, and charge are fundamentally different from one another. The units of a general physical quantity, such as force or capacitance, can always be expressed as some appropriate power law combination of the four fundamental units, M, L, T, and Q. Equation (1.2) makes dimensional sense because the constant ϵ_0 possesses the units $M^{-1}L^{-3}T^2Q^2$. Likewise, the Biot-Savart law only makes dimensional sense because the constant μ_0 possesses the units MLQ^{-2}. On the other hand, Eq. (1.3) does not make much dimensional sense; *i.e.*, the right-hand side and the left-hand side appear to possess different units. In fact, we can only reconcile Eqs. (1.2) and (1.3) if we divide the right-hand side of (1.3) by some constant, $4\pi\epsilon_0$, say, with dimensions $M^{-1}L^{-3}T^2Q^2$, which happens to have the numerical value unity for the particular choice of units in the Gaussian scheme. Likewise, the Gaussian version of the Biot-Savart law contains a hidden constant with the numerical value unity which also possesses dimensions. It can be seen that the apparent simplicity of the equations of electrodynamics in the Gaussian scheme is only achieved at the expense of wrecking their dimensionality. This is, perhaps, the best reason of all for not using Gaussian units.